Pensieve header: Fixing the bug with R2c.

In[∘]:=

In[∘]:=
```
Once[<< KnotTheory`];
```

Utilities. The step function, algebraic numbers, canonical forms.

In[∘]:=
```
θ[x_] /; NumericQ[x] := UnitStep[x]
```

In[∘]:=
```
ω2[v_][p_] := Module[{q = Expand[p], n, c},
  If[q === 0, 0, c = Coefficient[q, ω, n = Exponent[q, ω]];
   c v^n + ω2[v][q - c (ω + ω^-1)^n]]];
```

In[∘]:=
```
sign[ℰ_] := Module[{n, d, v, p, rs, e, k},
  {n, d} = NumeratorDenominator[ℰ];
  {n, d} /= ω^(Exponent[n,ω]/2+Exponent[n,ω,Min]/2);
  p = Factor[ω2[v]@n * ω2[v]@d /. v → 4 u^2 - 2];
  rs = Solve[p == 0, u, Reals];
  If[rs === {}, Sign[p /. u → 0],
   rs = Union@(u /. rs);
   Sign[(-1)^(e=Exponent[p,u]) Coefficient[p, u, e]] + Sum[
     k = 0; While[(d = RootReduce[∂_{u,++k} p /. u → r]) == 0];
     If[EvenQ[k], 0, 2 Sign[d]] * θ[u - r],
     {r, rs}]
  ]
 ]
```

In[∘]:=
```
SetAttributes[B, Orderless];
CF[b_B] := RotateLeft[#, First@Ordering[#] - 1] & /@ DeleteCases[b, {}]
```

In[∘]:=
```
CF[ℰ_] := Module[{γs = Union@Cases[ℰ, γ_ | γ̄_, ∞]},
  Total[CoefficientRules[ℰ, γs] /. (ps_ → c_) :→ Factor[c] Times @@ γs^ps] ]
```

In[∘]:=
```
CF[{}] = {};
CF[𝒞_List] := Module[{γs = Union@Cases[𝒞, γ_, ∞], γ},
  CF /@ DeleteCases[0][
    RowReduce[Table[∂_γ r, {r, 𝒞}, {γ, γs}]].γs] ]
```

In[∘]:=
```
(ℰ_)* := ℰ /. {γ̄ → γ, γ → γ̄, ω → ω^-1, c_Complex :→ c*};
r_Rule⁺ := {r, r*}
```

```
In[•]:= RulesOf[γi_ + rest_.] := (γi → -rest)+;
CF[PQ[C_, q_]] := Module[{nC = CF[C]},
   PQ[nC, CF[q /. Union @@ RulesOf /@ nC]] ]
```

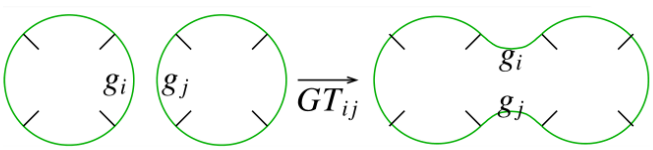
```
In[•]:= CF[Σb_[σ_, pq_]] := ΣCF[b][σ, CF[pq]]
```

Pretty-Printing.

```
In[•]:= Format[Σb_B[σ_, PQ[C_, q_]]] := Module[{γs},
    γs = γ# & /@ Join @@ b;
    Column[{TraditionalForm@σ,
      TableForm[Join[
        Prepend[""] /@ Table[TraditionalForm[∂c r], {r, C}, {c, γs}],
        {Prepend[""][
          Join @@ (b /. {l_, m___, r_} :→ {DisplayForm@RowBox[{"(", l}],
                m, DisplayForm@RowBox[{r, ")"}]}) /. i_Integer :→ γi ]},
        MapThread[Prepend, {Table[TraditionalForm[∂r,c q], {r, γs*}, {c, γs}], γs*}]
       ], TableAlignments → Center]
     }, Center] ];
```

The Face-Centric Core.

```
In[•]:= Σb1_[σ1_, PQ[C1_, q1_]] ⊕ Σb2_[σ2_, PQ[C2_, q2_]] ^:=
   CF@ΣJoin[b1,b2][σ1 + σ2, PQ[C1 ⋃ C2, q1 + q2]];
```



GT for Gap Touch:

```
In[•]:= GTi_,j_@ΣB[{li___,i_,ri___},{lj___,j_,rj___},bs___][σ_, PQ[C_, q_]] :=
  CF@ΣB[{ri,li,j,rj,lj,i},bs][σ, PQ[C ⋃ {γi - γj}, q]]
```

cor·don (kôr′dn)

n.

1. A line of people, military posts, or ships stationed around an area to enclose or guard it: *a police cordon.*

2. A rope, line, tape, or similar border stretched around an area, usually by the police, indicating that access is restricted.

\par\vskip 1mm\par\Cordon

In[∘]:=
```
Cordon_{i_}@Σ_B[{li___,i_,ri___},bs___][σ_, PQ[𝒞_, q_]] :=
 Module[{ϕ = ∂_{γ_i} 𝒞, λ = ∂_{γ̄_i,γ_i} q, nσ = σ, n𝒞, nq, p},
  {p} = FirstPosition[(# =!= 0) & /@ ϕ, True, {0}];
  {n𝒞, nq} = Which[
    p > 0, {𝒞, q} /. (γ_i → -𝒞⟦p⟧ / ϕ⟦p⟧)^+ /. (γ_i → 0)^+,
    λ =!= 0, (nσ += sign[λ]; {𝒞, q /. (γ_i → -(∂_{γ̄_i} q) / λ)^+ /. (γ_i → 0)^+}),
    λ === 0, {𝒞 ⋃ {∂_{γ̄_i} q}, q /. (γ_i → 0)^+}];
  CF@Σ_B[Most@{ri,li},bs][nσ, PQ[n𝒞, nq] /. (γ_{Last@{ri,li}} → γ_{First@{ri,li}})^+] ]
```

Strand Operations. *c* for contract, mc for magnetic contract:

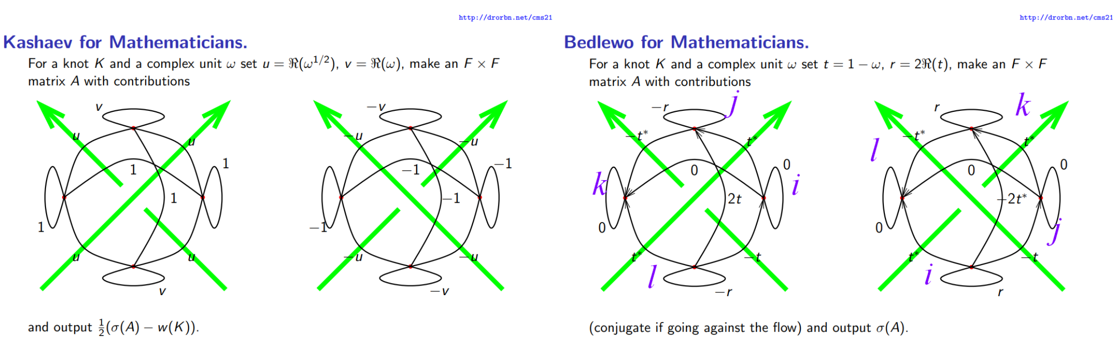
In[∘]:=
```
c_{i_,j_}@t : Σ_B[{li___,i_,ri___},{___,j_,___},___][__] := t // GT_{j,First@{ri,li}} // Cordon_j
```

In[∘]:=
```
c_{i_,j_}@t : Σ_B[{___,i_,j_,___},___][__] := Cordon_j@t
c_{i_,j_}@t : Σ_B[{j_,___,i_},___][__] := Cordon_j@t
c_{i_,j_}@t : Σ_B[{___,j_,i_,___},___][__] := Cordon_i@t
c_{i_,j_}@t : Σ_B[{i_,___,j_},___][__] := Cordon_i@t
```

In[∘]:=
```
mc[ℰ_] := ℰ //.
  t : Σ_B[{___,i_,___},{___,j_,___},___][__] | Σ_B[{___,i_,j_,___},___][__] | Σ_B[{j_,___,i_},___][__] /;
    i + j == 0 ⧴ c_{i,j}@t
```

The Crossings (and empty strands).

In[∘]:=
```
Kas@P_{i_,j_} := CF@Σ_B[{i,j}][0, PQ[{}, 0]];
TL@P_{i_,j_} := CF@Σ_B[{i,j}][0, PQ[{}, 0]]
```



In[∘]:=
```
Kas[x : X[i_, j_, k_, l_]] := Kas@If[PositiveQ[x], X_{-i,j,k,-l}, X̄_{-j,k,l,-i}];
Kas[(x : X | X̄)_{fs__}] := Module[{v = 2 u^2 - 1, p, γs, m},
  γs = γ_# & /@ {fs}; p = (x === X);
  m = If[p, ( v u 1 u
              u 1 u 1
              1 u v u
              u 1 u 1 ), - ( v u 1 u
                              u 1 u 1
                              1 u v u
                              u 1 u 1 )];
  CF@Σ_{B[{fs}]}[If[p, -1, 1], PQ[{}, γs*.m.γs]]]
```

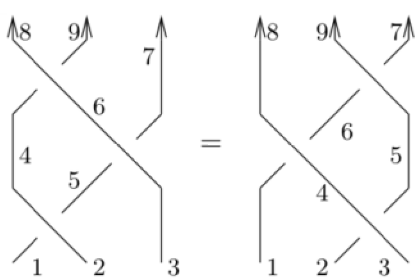
In[∘]:=
```
TL[x : X[i_, j_, k_, l_]] := TL@If[PositiveQ[x], X_{-i,j,k,-l}, X̄_{-j,k,l,-i}];
TL[(x : X | X̄)_{fs__}] := Module[{t = 1 - ω, r, γs, m},
  r = t + t*; γs = γ_# & /@ {fs};
  m = If[x === X,
    ( -r   -t  2 t  t*
      -t*   0  t*   0
      2 t*  t  -r  -t*
      t     0  -t   0 ), ( r    -t  -2 t*  t*
                           -t*   0   t*    0
                           -2 t  t   r     -t*
                           t     0   -t    0 )];
  CF@Σ_{B[{fs}]}[0, PQ[{}, γs*.m.γs]]]
```

\par{\bf\red Evaluation on Tangles and Knots.}

In[∘]:=
```
Kas[K_] := Fold[mc[#1 ⊕ #2] &, Σ_{B[]}[0, PQ[{}, 0]], List @@ (Kas /@ PD@K)];
KasSig[K_] := Expand[Kas[K][[1]] / 2]
```

In[∘]:=
```
TL[K_] := Fold[mc[#1 ⊕ #2] &, Σ_{B[]}[0, PQ[{}, 0]], List @@ (TL /@ PD@K)] /.
   θ[c_ + u] /; Abs[c] ≥ 1 :> θ[c];
TLSig[K_] := TL[K][[1]]
```

Reidemeister 3



In[∘]:=
```
R3L = PD[X_{-2,5,4,-1}, X_{-3,7,6,-5},
    X_{-6,9,8,-4}];
R3R = PD[X_{-3,5,4,-2}, X_{-4,6,8,-1},
    X_{-5,7,9,-6}];
{TL@R3L == TL@R3R, Kas@R3L == Kas@R3R}
```
Out[∘]=

{True, True}

In[∘]:=
```
TL@R3L
```
Out[∘]=

$$-1$$

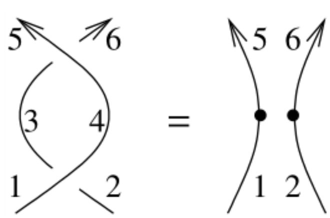
$$\begin{array}{c|cccccc} & \gamma_{-3} & \gamma_{7} & \gamma_{9} & \gamma_{8} & \gamma_{-1} & \gamma_{-2} \\ \hline \bar\gamma_{-3} & \frac{\omega^2+1}{\omega} & \omega-1 & -2\omega & 2 & 0 & -\frac{\omega+1}{\omega} \\ \bar\gamma_{7} & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 & 0 & 0 \\ \bar\gamma_{9} & -\frac{2}{\omega} & 1-\omega & \frac{\omega^2+1}{\omega} & -\frac{\omega+1}{\omega} & 0 & \frac{2}{\omega} \\ \bar\gamma_{8} & 2 & 0 & -\omega-1 & \frac{\omega^2+1}{\omega} & -\frac{\omega-1}{\omega} & -\frac{2}{\omega} \\ \bar\gamma_{-1} & 0 & 0 & 0 & \omega-1 & 0 & 1-\omega \\ \bar\gamma_{-2} & -\omega-1 & 0 & 2\omega & -2\omega & \frac{\omega-1}{\omega} & \frac{\omega^2+1}{\omega} \end{array}$$

In[∘]:=
```
Kas@R3L
```
Out[∘]=

$$2\,\Theta\left(u-\frac{1}{2}\right)-2\,\Theta\left(u+\frac{1}{2}\right)-2$$

$$\begin{array}{c|cccccc} & \gamma_{-3} & \gamma_{7} & \gamma_{9} & \gamma_{8} & \gamma_{-1} & \gamma_{-2} \\ \hline \bar\gamma_{-3} & \frac{2u^2(4u^2-3)}{(2u-1)(2u+1)} & \frac{u(4u^2-3)}{(2u-1)(2u+1)} & -\frac{1}{(2u-1)(2u+1)} & -\frac{2u}{(2u-1)(2u+1)} & -\frac{1}{(2u-1)(2u+1)} & \frac{u(4u^2-}{(2u-1)(2} \\ \bar\gamma_{7} & \frac{u(4u^2-3)}{(2u-1)(2u+1)} & \frac{2(2u^2-1)}{(2u-1)(2u+1)} & \frac{u(4u^2-3)}{(2u-1)(2u+1)} & -\frac{1}{(2u-1)(2u+1)} & -\frac{2u}{(2u-1)(2u+1)} & -\frac{1}{(2u-1)(} \\ \bar\gamma_{9} & -\frac{1}{(2u-1)(2u+1)} & \frac{u(4u^2-3)}{(2u-1)(2u+1)} & \frac{2u^2(4u^2-3)}{(2u-1)(2u+1)} & \frac{u(4u^2-3)}{(2u-1)(2u+1)} & -\frac{1}{(2u-1)(2u+1)} & -\frac{2u}{(2u-1)(} \\ \bar\gamma_{8} & -\frac{2u}{(2u-1)(2u+1)} & -\frac{1}{(2u-1)(2u+1)} & \frac{u(4u^2-3)}{(2u-1)(2u+1)} & \frac{2u^2(4u^2-3)}{(2u-1)(2u+1)} & \frac{u(4u^2-3)}{(2u-1)(2u+1)} & -\frac{1}{(2u-1)(} \\ \bar\gamma_{-1} & -\frac{1}{(2u-1)(2u+1)} & -\frac{2u}{(2u-1)(2u+1)} & -\frac{1}{(2u-1)(2u+1)} & \frac{u(4u^2-3)}{(2u-1)(2u+1)} & \frac{2(2u^2-1)}{(2u-1)(2u+1)} & \frac{u(4u^2-}{(2u-1)(2} \\ \bar\gamma_{-2} & \frac{u(4u^2-3)}{(2u-1)(2u+1)} & -\frac{1}{(2u-1)(2u+1)} & -\frac{2u}{(2u-1)(2u+1)} & -\frac{1}{(2u-1)(2u+1)} & \frac{u(4u^2-3)}{(2u-1)(2u+1)} & \frac{2u^2(4u^2}{(2u-1)(2} \end{array}$$

Reidemeister 2b



In[∘]:= $\mathrm{TL@PD}\left[X_{-2,4,3,-1},\ \overline{X}_{-4,6,5,-3}\right]$

Out[∘]=

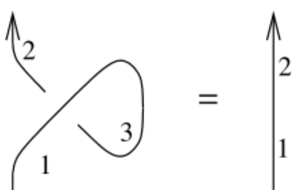
$$\begin{array}{c|cccc} & 0 & 0 & -1 & 0 \\ & 1 & 0 & -1 & 0 \\ & (\curlyvee_{-2} & \curlyvee_{6} & \curlyvee_{5} & \curlyvee_{-1}) \\ \overline{\curlyvee}_{-2} & 0 & 0 & 0 & 0 \\ \overline{\curlyvee}_{6} & 0 & 0 & 0 & 0 \\ \overline{\curlyvee}_{5} & 0 & 0 & 0 & 0 \\ \overline{\curlyvee}_{-1} & 0 & 0 & 0 & 0 \end{array}$$

In[∘]:= $\{\mathrm{TL@PD}\left[X_{-2,4,3,-1},\ \overline{X}_{-4,6,5,-3}\right] == \mathrm{GT}_{5,-2}\mathrm{@TL@PD}[P_{-1,5},\ P_{-2,6}],$
$\mathrm{Kas@PD}\left[X_{-2,4,3,-1},\ \overline{X}_{-4,6,5,-3}\right] == \mathrm{GT}_{5,-2}\mathrm{@Kas@PD}[P_{-1,5},\ P_{-2,6}]\}$

Out[∘]=

{True, True}

Reidemeister 1

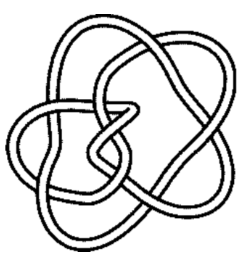


In[∘]:= $\{\mathrm{TL@PD}[X_{-3,3,2,-1}] == \mathrm{TL@P}_{-1,2},$
$\mathrm{Kas@PD}[X_{-3,3,2,-1}] == \mathrm{Kas@P}_{-1,2}\}$

Out[∘]=

{True, True}

A Knot



In[∘]:= f = TLSig[Knot[8, 5]]

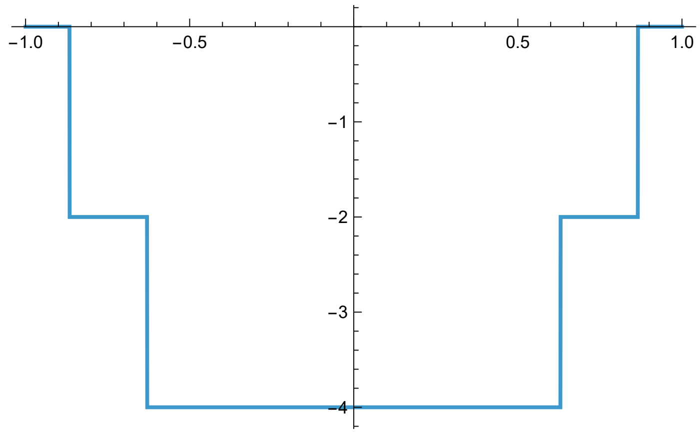
KnotTheory: Loading precomputed data in PD4Knots`.

Out[∘]=

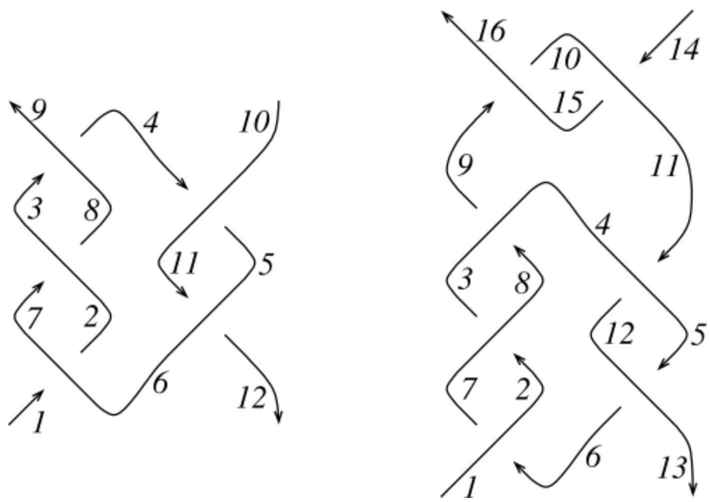
$$2\,\theta\left[-\frac{\sqrt{3}}{2}+u\right]-2\,\theta\left[\frac{\sqrt{3}}{2}+u\right]-2\,\theta\left[u-(\text{-0.630...})\right]+2\,\theta\left[u-(\text{0.630...})\right]$$

In[∘]:= **Plot[f, {u, -1, 1}]**

Out[∘]=



Some Tangles



In[∘]:=
$$\mathbf{T1} = \mathbf{PD}\left[\overline{X}_{-6,2,7,-1}, \overline{X}_{-2,8,3,-7}, \overline{X}_{-8,4,9,-3}, X_{-11,6,12,-5}, X_{-4,11,5,-10}\right];$$
$$\mathbf{T2} = \mathbf{PD}\left[X_{-6,2,7,-1}, X_{-2,8,3,-7}, X_{-8,4,9,-3}, \overline{X}_{-12,6,13,-5}, \overline{X}_{-4,12,5,-11}, \overline{X}_{-10,15,11,-14}, \overline{X}_{-15,10,16,-9}\right];$$

In[∘]:= **Column@{TL[T1], Kas[T1]}**

Out[∘]=

$$-2\,\theta\left(u-\frac{\sqrt{3}}{2}\right)+2\,\theta\left(u+\frac{\sqrt{3}}{2}\right)-1$$

$$\begin{array}{c|cccc}
 & (\Upsilon_{-10} & \Upsilon_{9} & \Upsilon_{-1} & \Upsilon_{12}) \\
\hline
\bar\Upsilon_{-10} & 0 & 1-\omega & 0 & \omega-1 \\
\bar\Upsilon_{9} & \frac{\omega-1}{\omega} & \frac{2\,\omega}{\omega^2-\omega+1} & -\frac{\omega-1}{\omega} & -\frac{2\,\omega}{\omega^2-\omega+1} \\
\bar\Upsilon_{-1} & 0 & \omega-1 & 0 & 1-\omega \\
\bar\Upsilon_{12} & -\frac{\omega-1}{\omega} & -\frac{2\,\omega}{\omega^2-\omega+1} & \frac{\omega-1}{\omega} & \frac{2\,\omega}{\omega^2-\omega+1}
\end{array}$$

$$-2\,\theta\left(u-\frac{\sqrt{3}}{2}\right)+2\,\theta\left(u+\frac{\sqrt{3}}{2}\right)-1$$

$$\begin{array}{c|cccc}
 & (\Upsilon_{-10} & \Upsilon_{9} & \Upsilon_{-1} & \Upsilon_{12}) \\
\hline
\bar\Upsilon_{-10} & 2\,(u-1)\,(u+1)\,(4\,u^2-3) & 0 & -2\,(u-1)\,(u+1)\,(4\,u^2-3) & 0 \\
\bar\Upsilon_{9} & 0 & \frac{1}{2\,(4\,u^2-3)} & 0 & -\frac{1}{2\,(4\,u^2-3)} \\
\bar\Upsilon_{-1} & -2\,(u-1)\,(u+1)\,(4\,u^2-3) & 0 & 2\,(u-1)\,(u+1)\,(4\,u^2-3) & 0 \\
\bar\Upsilon_{12} & 0 & -\frac{1}{2\,(4\,u^2-3)} & 0 & \frac{1}{2\,(4\,u^2-3)}
\end{array}$$

In[∘]:= **Column@{TL[T2], Kas[T2]}**

Out[∘]=

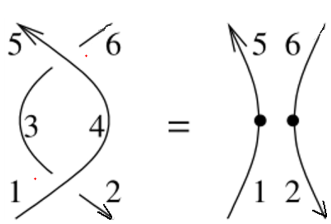
$$0$$

$$\begin{array}{c|cccc}
 & (\Upsilon_{-14} & \Upsilon_{16} & \Upsilon_{-1} & \Upsilon_{13}) \\
\hline
\bar\Upsilon_{-14} & 0 & 1-\omega & 0 & \omega-1 \\
\bar\Upsilon_{16} & \frac{\omega-1}{\omega} & -\frac{2\,(\omega-1)^2\,\omega}{\omega^4-3\,\omega^3+5\,\omega^2-3\,\omega+1} & -\frac{\omega-1}{\omega} & \frac{2\,(\omega-1)^2\,\omega}{\omega^4-3\,\omega^3+5\,\omega^2-3\,\omega+1} \\
\bar\Upsilon_{-1} & 0 & \omega-1 & 0 & 1-\omega \\
\bar\Upsilon_{13} & -\frac{\omega-1}{\omega} & \frac{2\,(\omega-1)^2\,\omega}{\omega^4-3\,\omega^3+5\,\omega^2-3\,\omega+1} & \frac{\omega-1}{\omega} & -\frac{2\,(\omega-1)^2\,\omega}{\omega^4-3\,\omega^3+5\,\omega^2-3\,\omega+1}
\end{array}$$

$$1$$

$$\begin{array}{c|cccc}
 & (\Upsilon_{-14} & \Upsilon_{16} & \Upsilon_{-1} & \Upsilon_{13}) \\
\hline
\bar\Upsilon_{-14} & \frac{1}{2}\left(-16\,u^4+28\,u^2-13\right) & 0 & \frac{1}{2}\left(16\,u^4-28\,u^2+13\right) & 0 \\
\bar\Upsilon_{16} & 0 & -\frac{2\,(u-1)\,(u+1)}{16\,u^4-28\,u^2+13} & 0 & \frac{2\,(u-1)\,(u+1)}{16\,u^4-28\,u^2+13} \\
\bar\Upsilon_{-1} & \frac{1}{2}\left(16\,u^4-28\,u^2+13\right) & 0 & \frac{1}{2}\left(-16\,u^4+28\,u^2-13\right) & 0 \\
\bar\Upsilon_{13} & 0 & \frac{2\,(u-1)\,(u+1)}{16\,u^4-28\,u^2+13} & 0 & -\frac{2\,(u-1)\,(u+1)}{16\,u^4-28\,u^2+13}
\end{array}$$

Reidemeister 2c



In[∘]:= **Kas@PD[$\bar{X}_{-1,2,4,-3}$, $X_{-6,5,3,-4}$] == $GT_{5,2}$@Kas@PD[$P_{-1,5}$, $P_{-6,2}$]**

Out[∘]=

True

In[∘]:= $\{\mathrm{TL@PD}[\overline{X}_{-1,2,4,-3}, X_{-6,5,3,-4}], \mathrm{TL@PD}[X_{-1,2,4,-3}, \overline{X}_{-6,5,3,-4}]\}$

Out[∘]=

$$\left\{\begin{array}{c|cccc} & & 0 & & \\ & (\gamma_{-6} & \gamma_5 & \gamma_{-1} & \gamma_2) \\ \bar\gamma_{-6} & 0 & \omega-1 & 0 & 1-\omega \\ \bar\gamma_5 & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 \\ \bar\gamma_{-1} & 0 & 1-\omega & 0 & \omega-1 \\ \bar\gamma_2 & \frac{\omega-1}{\omega} & 0 & -\frac{\omega-1}{\omega} & 0 \end{array}, \begin{array}{c|cccc} & & 0 & & \\ & (\gamma_{-6} & \gamma_5 & \gamma_{-1} & \gamma_2) \\ \bar\gamma_{-6} & 0 & \omega-1 & 0 & 1-\omega \\ \bar\gamma_5 & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 \\ \bar\gamma_{-1} & 0 & 1-\omega & 0 & \omega-1 \\ \bar\gamma_2 & \frac{\omega-1}{\omega} & 0 & -\frac{\omega-1}{\omega} & 0 \end{array}\right\}$$

In[∘]:= $\mathrm{TL@PD}[P_{-1,5}, P_{-6,2}]$

Out[∘]=

$$\begin{array}{c|cccc} & & 0 & & \\ & (\gamma_{-6} & \gamma_2) & (\gamma_{-1} & \gamma_5) \\ \bar\gamma_{-6} & 0 & 0 & 0 & 0 \\ \bar\gamma_2 & 0 & 0 & 0 & 0 \\ \bar\gamma_{-1} & 0 & 0 & 0 & 0 \\ \bar\gamma_5 & 0 & 0 & 0 & 0 \end{array}$$

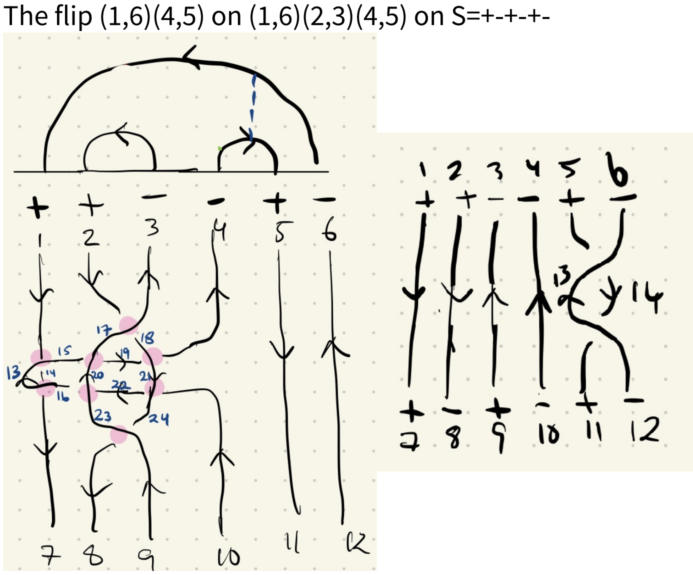
In[∘]:= $\mathrm{TL@PD}[\overline{X}_{-1,2,4,-3}, X_{-6,5,3,-4}] == \mathrm{GT}_{5,2}\mathrm{@TL@PD}[P_{-1,5}, P_{-6,2}]$

Out[∘]=

$$\begin{array}{c|cccc} & & 0 & & \\ & (\gamma_{-6} & \gamma_5 & \gamma_{-1} & \gamma_2) \\ \bar\gamma_{-6} & 0 & \omega-1 & 0 & 1-\omega \\ \bar\gamma_5 & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 \\ \bar\gamma_{-1} & 0 & 1-\omega & 0 & \omega-1 \\ \bar\gamma_2 & \frac{\omega-1}{\omega} & 0 & -\frac{\omega-1}{\omega} & 0 \end{array} == \begin{array}{c|cccc} & & 0 & & \\ & 0 & -1 & 0 & 1 \\ & (\gamma_{-6} & \gamma_5 & \gamma_{-1} & \gamma_2) \\ \bar\gamma_{-6} & 0 & 0 & 0 & 0 \\ \bar\gamma_5 & 0 & 0 & 0 & 0 \\ \bar\gamma_{-1} & 0 & 0 & 0 & 0 \\ \bar\gamma_2 & 0 & 0 & 0 & 0 \end{array}$$

That’s a fail for TL!

The flip (1,6)(4,5) on (1,6)(2,3)(4,5) on S=+-+-+-



In[∘]:= $\mathrm{arc15flip1} = \mathrm{PD}[P_{-5,11}, P_{-12,6}, \overline{X}_{-13,14,15,-1}, X_{-14,13,7,-16},$
$\overline{X}_{-17,18,3,-2}, X_{-24,23,8,-9}, \overline{X}_{-20,19,17,-15}, X_{-19,21,4,-18}, X_{-22,20,16,-23}, \overline{X}_{-21,22,24,-10}];$
$\mathrm{arc15flip2} = \mathrm{PD}[P_{-1,7}, P_{-2,8}, P_{-9,3}, P_{-10,4}, \overline{X}_{-13,14,6,-5}, X_{-14,13,11,-12}];$

In[•]:= $GT_{4,11}$@$GT_{-5,-12}$@TL@arc15flip1

Out[•]=

$$0$$

$$\begin{array}{c|cccccccccc}
 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \\
 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & -1 & \\
 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \\
 & (\gamma_{-12} & \gamma_{6} & \gamma_{-5} & \gamma_{4} & \gamma_{3} & \gamma_{-2} & \gamma_{-1} & \gamma_{7} & \gamma_{8} & \gamma \\
\bar\gamma_{-12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
\bar\gamma_{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
\bar\gamma_{-5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
\bar\gamma_{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
\bar\gamma_{3} & 0 & 0 & 0 & 0 & 0 & -\frac{\omega-1}{\omega} & \frac{(\omega-1)(\omega+1)}{\omega} & 0 & -\frac{(\omega-1)(\omega+1)}{\omega} & \omega \\
\bar\gamma_{-2} & 0 & 0 & 0 & 0 & \omega-1 & 0 & 1-\omega & 0 & 0 & \\
\bar\gamma_{-1} & 0 & 0 & 0 & 0 & -\frac{(\omega-1)(\omega+1)}{\omega} & \frac{\omega-1}{\omega} & 0 & 1-\omega & \frac{(\omega-1)(\omega+1)}{\omega} & - \\
\bar\gamma_{7} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\omega-1}{\omega} & 0 & -\frac{\omega-1}{\omega} & \\
\bar\gamma_{8} & 0 & 0 & 0 & 0 & \frac{(\omega-1)(\omega+1)}{\omega} & 0 & -\frac{(\omega-1)(\omega+1)}{\omega} & \omega-1 & 0 & \\
\bar\gamma_{-9} & 0 & 0 & 0 & 0 & 1-\omega & 0 & \omega-1 & 0 & 0 & \\
\bar\gamma_{-10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
\bar\gamma_{11} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} &
\end{array}$$

In[•]:= $c_{6,-1}$@$c_{3,-2}$@$c_{4,-5}$@$GT_{4,11}$@$GT_{-5,-12}$@TL@arc15flip1

Out[•]=

$$0$$

$$\begin{array}{c|cccccc}
 & 0 & 0 & -1 & 0 & 1 & 0 \\
 & (\gamma_{-12} & \gamma_{7} & \gamma_{8} & \gamma_{-9} & \gamma_{-10} & \gamma_{11}) \\
\bar\gamma_{-12} & 0 & 1-\omega & 0 & 0 & 0 & \omega-1 \\
\bar\gamma_{7} & \frac{\omega-1}{\omega} & 0 & -\frac{\omega-1}{\omega} & 0 & 0 & 0 \\
\bar\gamma_{8} & 0 & \omega-1 & 0 & 0 & 0 & 1-\omega \\
\bar\gamma_{-9} & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar\gamma_{-10} & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar\gamma_{11} & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 & 0 & 0
\end{array}$$

In[∘]:= **$GT_{4,11}$@$GT_{3,-10}$@$GT_{-2,-9}$@$GT_{-1,8}$@TL@arc15flip2**

Out[∘]=

$$0$$

$$\begin{array}{c|cccccccccccc}
 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
 & (\Upsilon_{-12} & \Upsilon_{6} & \Upsilon_{-5} & \Upsilon_{4} & \Upsilon_{3} & \Upsilon_{-2} & \Upsilon_{-1} & \Upsilon_{7} & \Upsilon_{8} & \Upsilon_{-9} & \Upsilon_{-10} & \Upsilon_{11}) \\
\bar{\Upsilon}_{-12} & 0 & 1-\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega-1 \\
\bar{\Upsilon}_{6} & \frac{\omega-1}{\omega} & 0 & -\frac{\omega-1}{\omega} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar{\Upsilon}_{-5} & 0 & \omega-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-\omega \\
\bar{\Upsilon}_{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar{\Upsilon}_{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar{\Upsilon}_{-2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar{\Upsilon}_{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar{\Upsilon}_{7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar{\Upsilon}_{8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar{\Upsilon}_{-9} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar{\Upsilon}_{-10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar{\Upsilon}_{11} & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

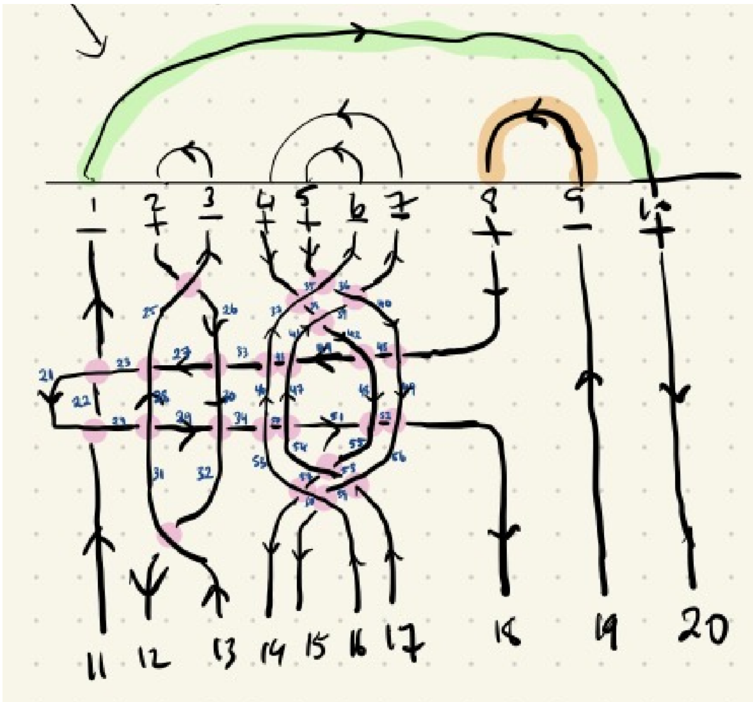
In[∘]:= **$c_{6,-1}$@$c_{3,-2}$@$c_{4,-5}$@$GT_{4,11}$@$GT_{3,-10}$@$GT_{-2,-9}$@$GT_{-1,8}$@TL@arc15flip2**

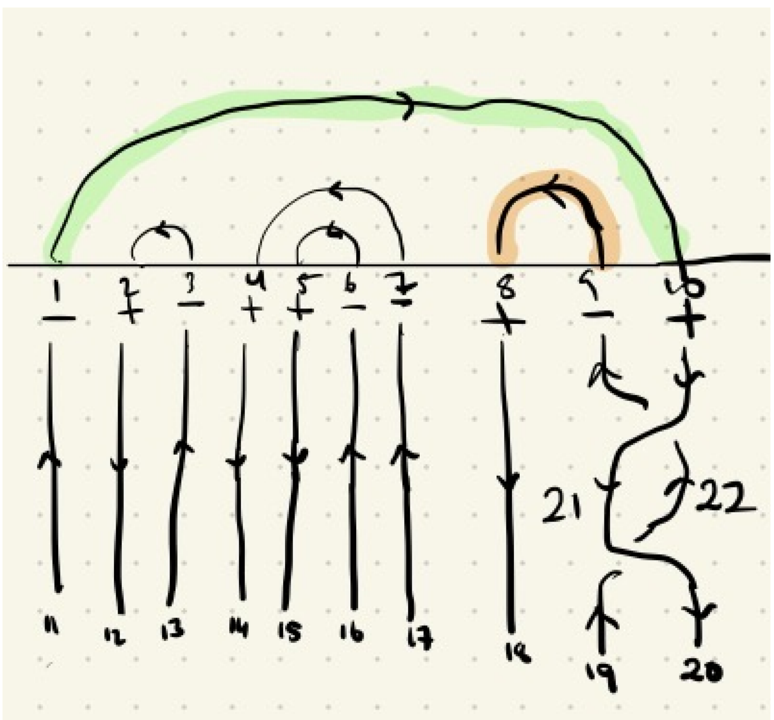
Out[∘]=

$$0$$

$$\begin{array}{c|cccccc}
 & 0 & 0 & -1 & 0 & 1 & 0 \\
 & (\Upsilon_{-12} & \Upsilon_{7} & \Upsilon_{8} & \Upsilon_{-9} & \Upsilon_{-10} & \Upsilon_{11}) \\
\bar{\Upsilon}_{-12} & 0 & 1-\omega & 0 & 0 & 0 & \omega-1 \\
\bar{\Upsilon}_{7} & \frac{\omega-1}{\omega} & 0 & -\frac{\omega-1}{\omega} & 0 & 0 & 0 \\
\bar{\Upsilon}_{8} & 0 & \omega-1 & 0 & 0 & 0 & 1-\omega \\
\bar{\Upsilon}_{-9} & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar{\Upsilon}_{-10} & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar{\Upsilon}_{11} & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 & 0 & 0
\end{array}$$

Wrong orientation corridor flip on S = -+-++--+-+, (1,10)(2,3)(4,7)(5,6)(8,9). (Flipping (1,10) and (8,9)).





In[∘]:= **flip810a** = PD$\left[P_{-19,9}, P_{-10,20}, \overline{X}_{-23,1,21,-22}, X_{-11,24,22,-21}, \overline{X}_{-25,26,3,-2}, X_{-27,25,23,-28}, \overline{X}_{-26,27,30,-33},\right.$
$\overline{X}_{-31,29,28,-24}, X_{-29,32,34,-30}, X_{-32,31,12,-13}, \overline{X}_{-35,36,6,-5}, \overline{X}_{-37,38,35,-4}, X_{-39,40,7,-36}, \overline{X}_{-41,42,39,-38},$
$X_{-59,60,15,-16}, X_{-57,53,14,-60}, \overline{X}_{-56,58,59,-17}, X_{-55,54,57,-58}, X_{-43,37,33,-46}, X_{-44,41,43,-47},$
$\left.\overline{X}_{-53,50,46,-34}, \overline{X}_{-54,51,47,-50}, \overline{X}_{-42,44,48,-45}, \overline{X}_{-40,45,49,-8}, X_{-51,55,52,-48}, X_{-52,56,18,-49}\right]$;

flip810b = PD$\left[P_{-11,1}, P_{-2,12}, P_{-13,3}, P_{-4,14}, P_{-5,15}, P_{-16,6}, P_{-17,7}, P_{-8,18}, \overline{X}_{-10,9,21,-22}, X_{-19,20,22,-21}\right]$;

In[∘]:= **$GT_{9,20}$@$GT_{-8,-19}$@TL@flip810a**

Out[∘]=

										0	
	1	0	0	0	−1	0	0	0	0	0	0
	0	0	0	0	0	0	−1	0	1	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	1	0
	0	0	0	0	0	1	0	0	0	0	0
	0	−1	0	1	0	0	0	0	0	0	0
	(Υ_{-19}	Υ_{20}	Υ_{-10}	Υ_{9}	Υ_{-8}	Υ_{7}	Υ_{6}	Υ_{-5}	Υ_{-4}	Υ_{3}	Υ_{-2}
$\overline{\Upsilon}_{-19}$	0	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{20}$	0	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{-10}$	0	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{9}$	0	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{-8}$	0	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{7}$	0	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{6}$	0	0	0	0	0	0	0	$-\frac{\omega-1}{\omega}$	$\frac{(\omega-1)(\omega+1)}{\omega}$	0	0
$\overline{\Upsilon}_{-5}$	0	0	0	0	0	0	$\omega-1$	0	$1-\omega$	0	0
$\overline{\Upsilon}_{-4}$	0	0	0	0	0	0	$-\frac{(\omega-1)(\omega+1)}{\omega}$	$\frac{\omega-1}{\omega}$	0	0	0
$\overline{\Upsilon}_{3}$	0	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{-2}$	0	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{1}$	0	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{-11}$	0	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{12}$	0	0	0	0	0	0	0	0	0	0	$\frac{\omega-1}{\omega}$
$\overline{\Upsilon}_{-13}$	0	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{14}$	0	0	0	0	0	0	0	0	$\frac{\omega-1}{\omega}$	0	$-\frac{\omega-1}{\omega}$
$\overline{\Upsilon}_{15}$	0	0	0	0	0	0	$\frac{(\omega-1)(\omega+1)}{\omega}$	0	$-\frac{(\omega-1)(\omega+1)}{\omega}$	0	0
$\overline{\Upsilon}_{-16}$	0	0	0	0	0	0	$1-\omega$	0	$\omega-1$	0	0
$\overline{\Upsilon}_{-17}$	0	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{18}$	0	0	0	0	0	0	0	0	$-\frac{\omega-1}{\omega}$	0	0

In[∘]:= **$c_{1,-10}$@$c_{-4,7}$@$c_{-5,6}$@$c_{-8,9}$@$c_{-2,3}$@$GT_{9,20}$@$GT_{-8,-19}$@TL@flip810a**

Out[∘]=

					0					
	0	0	0	0	0	0	−1	0	1	0
	0	−1	0	1	0	0	0	0	0	0
	0	−1	0	0	0	1	0	0	0	0
	0	−1	0	0	0	0	0	0	0	1
	(Υ_{-19}	Υ_{20}	Υ_{-11}	Υ_{12}	Υ_{-13}	Υ_{14}	Υ_{15}	Υ_{-16}	Υ_{-17}	Υ_{18})
$\overline{\Upsilon}_{-19}$	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{20}$	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{-11}$	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{12}$	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{-13}$	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{14}$	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{15}$	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{-16}$	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{-17}$	0	0	0	0	0	0	0	0	0	0
$\overline{\Upsilon}_{18}$	0	0	0	0	0	0	0	0	0	0

In[∘]:= $GT_{-8,-19}$@$GT_{7,18}$@$GT_{6,-17}$@$GT_{-5,-16}$@$GT_{-4,15}$@$GT_{3,14}$@$GT_{-2,-13}$@$GT_{1,12}$@TL@flip810b

Out[∘]=

0

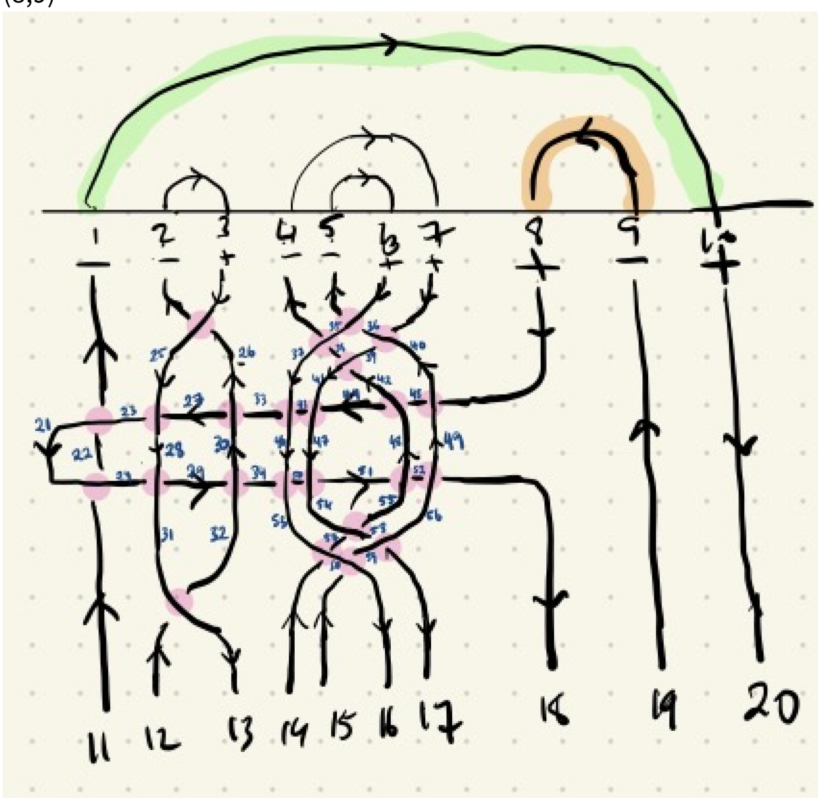
	1	0	0	0	−1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	−1	0	0	0	0	0	0
	0	0	0	0	0	0	0	−1	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	−1	0	0
	0	0	0	0	0	0	0	0	1	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	1	0
	0	0	0	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	1	0	0	0	0	0	0	0
	(γ_{-19}	γ_{20}	γ_{-10}	γ_{9}	γ_{-8}	γ_{7}	γ_{6}	γ_{-5}	γ_{-4}	γ_{3}	γ_{-2}	γ_{1}	γ_{-}
$\bar{\gamma}_{-19}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{\gamma}_{20}$	0	0	$\frac{\omega-1}{\omega}$	0	$-\frac{\omega-1}{\omega}$	0	0	0	0	0	0	0	0
$\bar{\gamma}_{-10}$	0	$1-\omega$	0	$\omega-1$	0	0	0	0	0	0	0	0	0
$\bar{\gamma}_{9}$	0	0	$-\frac{\omega-1}{\omega}$	0	$\frac{\omega-1}{\omega}$	0	0	0	0	0	0	0	0
$\bar{\gamma}_{-8}$	0	$\omega-1$	0	$1-\omega$	0	0	0	0	0	0	0	0	0
$\bar{\gamma}_{7}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{\gamma}_{6}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{\gamma}_{-5}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{\gamma}_{-4}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{\gamma}_{3}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{\gamma}_{-2}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{\gamma}_{1}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{\gamma}_{-11}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{\gamma}_{12}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{\gamma}_{-13}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{\gamma}_{14}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{\gamma}_{15}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{\gamma}_{-16}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{\gamma}_{-17}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{\gamma}_{18}$	0	0	0	0	0	0	0	0	0	0	0	0	0

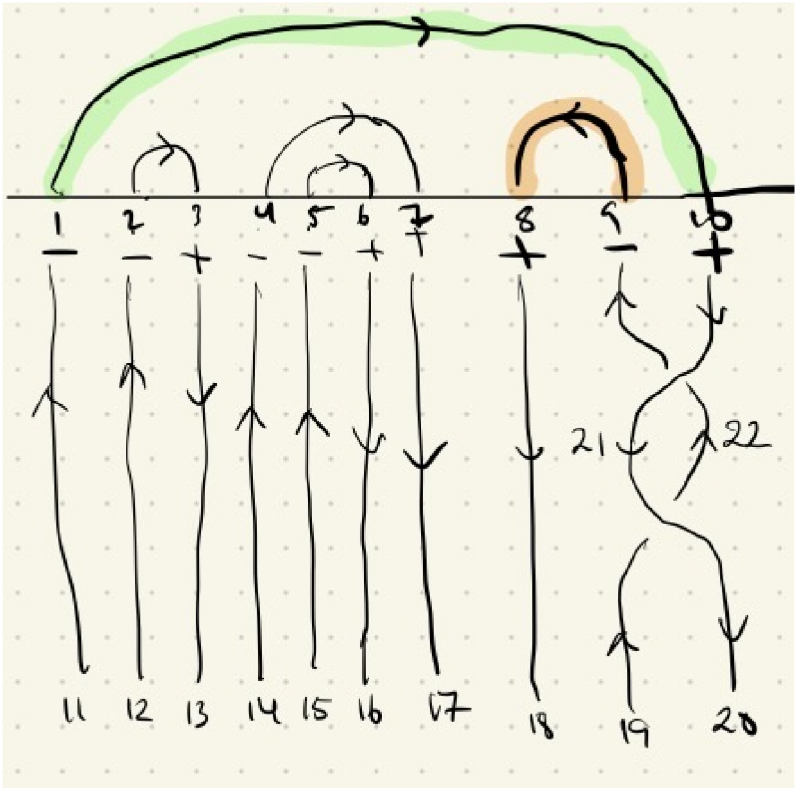
In[∘]:= $\mathbf{c}_{1,-10}$@$\mathbf{c}_{-4,7}$@
$\mathbf{c}_{-5,6}$@$\mathbf{c}_{-8,9}$@$\mathbf{c}_{-2,3}$@$\mathbf{GT}_{-8,-19}$@$\mathbf{GT}_{7,18}$@$\mathbf{GT}_{6,-17}$@$\mathbf{GT}_{-5,-16}$@$\mathbf{GT}_{-4,15}$@$\mathbf{GT}_{3,14}$@$\mathbf{GT}_{-2,-13}$@$\mathbf{GT}_{1,12}$@**TL**@**flip810b**

Out[∘]=

$$\begin{array}{c|cccccccccc}
 & & & & & 0 & & & & & \\
 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\
 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\
 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\
 & (\gamma_{-19} & \gamma_{20} & \gamma_{-11} & \gamma_{12} & \gamma_{-13} & \gamma_{14} & \gamma_{15} & \gamma_{-16} & \gamma_{-17} & \gamma_{18}) \\
\overline{\gamma}_{-19} & 0 & \omega-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-\omega \\
\overline{\gamma}_{20} & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\gamma}_{-11} & 0 & 1-\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega-1 \\
\overline{\gamma}_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\gamma}_{-13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\gamma}_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\gamma}_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\gamma}_{-16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\gamma}_{-17} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\gamma}_{18} & \frac{\omega-1}{\omega} & 0 & -\frac{\omega-1}{\omega} & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

Correct orientation corridor flip on S = --+--+++-+, (1,10)(2,3)(4,7)(5,6)(8,9) (flipping (1,10) and (8,9)





In[ ]:=

$$\text{flip810c} = \text{PD}\left[P_{-19,9},\, P_{-10,20},\, \overline{X}_{-23,1,21,-22},\, X_{-11,24,22,-21},\, \overline{X}_{-3,2,25,-26},\, \overline{X}_{-25,23,28,-27},\, X_{-33,26,27,-30},\, X_{-24,31,29,-28},\, \overline{X}_{-32,34,30,-29},\, X_{-12,13,32,-31},\, \overline{X}_{-6,5,35,-36},\, \overline{X}_{-35,4,37,-38},\, X_{-7,36,39,-40},\, \overline{X}_{-39,38,41,-42},\, X_{-57,58,55,-54},\, X_{-14,60,57,-53},\, \overline{X}_{-59,17,56,-58},\, X_{-15,16,59,-60},\, \overline{X}_{-37,33,46,-43},\, \overline{X}_{-41,43,47,-44},\, X_{-34,53,50,-46},\, X_{-50,54,51,-47},\, X_{-45,42,44,-48},\, X_{-8,40,45,-49},\, \overline{X}_{-55,52,48,-51},\, \overline{X}_{-56,18,49,-52}\right];$$

$$\text{flip810d} = \text{PD}\left[P_{-11,1},\, P_{-12,2},\, P_{-3,13},\, P_{-14,4},\, P_{-15,5},\, P_{-6,16},\, P_{-7,17},\, P_{-8,18},\, \overline{X}_{-10,9,21,-22},\, X_{-19,20,22,-21}\right];$$

In[∘]:= **GT$_{9,20}$@GT$_{-8,-19}$@TL@flip810c**

Out[∘]=

$$
\begin{array}{c|ccccccccc}
 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\
 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
 & (\curlyvee_{-19} & \curlyvee_{20} & \curlyvee_{-10} & \curlyvee_{9} & \curlyvee_{-8} & \curlyvee_{-7} & \curlyvee_{-6} & \curlyvee_{5} & \curlyvee_{4} \\
\overline{\curlyvee}_{-19} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\curlyvee}_{20} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\curlyvee}_{-10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\curlyvee}_{9} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\curlyvee}_{-8} & 0 & 0 & 0 & 0 & 0 & \omega-1 & 0 & 0 & 0 \\
\overline{\curlyvee}_{-7} & 0 & 0 & 0 & 0 & -\frac{\omega-1}{\omega} & 0 & 0 & 0 & \omega-1 \\
\overline{\curlyvee}_{-6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega-1 & -\frac{(\omega-1)(\omega+1)}{\omega} \\
\overline{\curlyvee}_{5} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} \\
\overline{\curlyvee}_{4} & 0 & 0 & 0 & 0 & 0 & -\frac{\omega-1}{\omega} & \frac{(\omega-1)(\omega+1)}{\omega} & 1-\omega & 0 \\
\overline{\curlyvee}_{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 1-\omega & 0 & 0 \\
\overline{\curlyvee}_{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\curlyvee}_{1} & 0 & 0 & 0 & 0 & 0 & \frac{(\omega-1)(\omega+1)}{\omega} & 0 & 0 & 0 \\
\overline{\curlyvee}_{-11} & 0 & 0 & 0 & 0 & 0 & 1-\omega & 0 & 0 & 0 \\
\overline{\curlyvee}_{-12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\curlyvee}_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\curlyvee}_{-14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\curlyvee}_{-15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\curlyvee}_{16} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\omega-1}{\omega} & 0 & -\frac{\omega-1}{\omega} \\
\overline{\curlyvee}_{17} & 0 & 0 & 0 & 0 & 0 & \frac{\omega-1}{\omega} & -\frac{(\omega-1)(\omega+1)}{\omega} & 0 & \frac{(\omega-1)(\omega+1)}{\omega} \\
\overline{\curlyvee}_{18} & 0 & 0 & 0 & 0 & \frac{\omega-1}{\omega} & -\frac{(\omega-1)(\omega+1)}{\omega} & \omega-1 & 0 & 1-\omega
\end{array}
$$

In[∘]:= **c$_{1,-10}$@c$_{9,-8}$@c$_{4,-7}$@c$_{5,-6}$@c$_{2,-3}$@GT$_{9,20}$@GT$_{-8,-19}$@TL@flip810c**

Out[∘]=

$$0$$

$$
\begin{array}{c|cccccccccc}
 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\
 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\
 & (\curlyvee_{-19} & \curlyvee_{20} & \curlyvee_{-11} & \curlyvee_{-12} & \curlyvee_{13} & \curlyvee_{-14} & \curlyvee_{-15} & \curlyvee_{16} & \curlyvee_{17} & \curlyvee_{18}) \\
\overline{\curlyvee}_{-19} & 0 & \omega-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-\omega \\
\overline{\curlyvee}_{20} & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\curlyvee}_{-11} & 0 & 1-\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega-1 \\
\overline{\curlyvee}_{-12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\curlyvee}_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\curlyvee}_{-14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\curlyvee}_{-15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\curlyvee}_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\curlyvee}_{17} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\curlyvee}_{18} & \frac{\omega-1}{\omega} & 0 & -\frac{\omega-1}{\omega} & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

In[∘]:= $\mathrm{GT}_{-8,-19}$@$\mathrm{GT}_{-7,18}$@$\mathrm{GT}_{-6,17}$@$\mathrm{GT}_{5,16}$@$\mathrm{GT}_{4,-15}$@$\mathrm{GT}_{-3,-14}$@$\mathrm{GT}_{2,13}$@$\mathrm{GT}_{1,-12}$@TL@flip810d

Out[∘]=

0

	1	0	0	0	−1	0	0	0	0	0	0	0	(
	0	0	0	0	0	0	0	0	−1	0	0	0	(
	0	0	0	0	0	0	0	0	0	−1	0	0	(
	0	0	0	0	0	0	0	0	0	0	0	−1	(
	0	0	0	0	0	1	0	0	0	0	0	0	(
	0	0	0	0	0	0	1	0	0	0	0	0	(
	0	0	0	0	0	0	0	0	0	0	1	0	(
	0	0	0	0	0	0	0	1	0	0	0	0	(
	(γ_{-19}	γ_{20}	γ_{-10}	γ_{9}	γ_{-8}	γ_{-7}	γ_{-6}	γ_{5}	γ_{4}	γ_{-3}	γ_{2}	γ_{1}	γ_{-}
$\bar{\gamma}_{-19}$	0	0	0	0	0	0	0	0	0	0	0	0	(
$\bar{\gamma}_{20}$	0	0	$\frac{\omega-1}{\omega}$	0	$-\frac{\omega-1}{\omega}$	0	0	0	0	0	0	0	(
$\bar{\gamma}_{-10}$	0	$1-\omega$	0	$\omega-1$	0	0	0	0	0	0	0	0	(
$\bar{\gamma}_{9}$	0	0	$-\frac{\omega-1}{\omega}$	0	$\frac{\omega-1}{\omega}$	0	0	0	0	0	0	0	(
$\bar{\gamma}_{-8}$	0	$\omega-1$	0	$1-\omega$	0	0	0	0	0	0	0	0	(
$\bar{\gamma}_{-7}$	0	0	0	0	0	0	0	0	0	0	0	0	(
$\bar{\gamma}_{-6}$	0	0	0	0	0	0	0	0	0	0	0	0	(
$\bar{\gamma}_{5}$	0	0	0	0	0	0	0	0	0	0	0	0	(
$\bar{\gamma}_{4}$	0	0	0	0	0	0	0	0	0	0	0	0	(
$\bar{\gamma}_{-3}$	0	0	0	0	0	0	0	0	0	0	0	0	(
$\bar{\gamma}_{2}$	0	0	0	0	0	0	0	0	0	0	0	0	(
$\bar{\gamma}_{1}$	0	0	0	0	0	0	0	0	0	0	0	0	(
$\bar{\gamma}_{-11}$	0	0	0	0	0	0	0	0	0	0	0	0	(
$\bar{\gamma}_{-12}$	0	0	0	0	0	0	0	0	0	0	0	0	(
$\bar{\gamma}_{13}$	0	0	0	0	0	0	0	0	0	0	0	0	(
$\bar{\gamma}_{-14}$	0	0	0	0	0	0	0	0	0	0	0	0	(
$\bar{\gamma}_{-15}$	0	0	0	0	0	0	0	0	0	0	0	0	(
$\bar{\gamma}_{16}$	0	0	0	0	0	0	0	0	0	0	0	0	(
$\bar{\gamma}_{17}$	0	0	0	0	0	0	0	0	0	0	0	0	(
$\bar{\gamma}_{18}$	0	0	0	0	0	0	0	0	0	0	0	0	(

In[∘]:=
$$c_{1,-10}@c_{9,-8}@c_{4,-7}@c_{5,-6}@c_{2,-3}@GT_{-8,-19}@GT_{-7,18}@GT_{-6,17}@GT_{5,16}@GT_{4,-15}@GT_{-3,-14}@GT_{2,13}@GT_{1,-12}@TL@\mathtt{flip810d}$$

Out[∘]=

$$0$$

$$\begin{array}{c|cccccccccc}
 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\
 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\
 & \gamma_{-19} & \gamma_{20} & \gamma_{-11} & \gamma_{-12} & \gamma_{13} & \gamma_{-14} & \gamma_{-15} & \gamma_{16} & \gamma_{17} & \gamma_{18} \\
\bar\gamma_{-19} & 0 & \omega-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-\omega \\
\bar\gamma_{20} & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar\gamma_{-11} & 0 & 1-\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega-1 \\
\bar\gamma_{-12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar\gamma_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar\gamma_{-14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar\gamma_{-15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar\gamma_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar\gamma_{17} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar\gamma_{18} & \frac{\omega-1}{\omega} & 0 & -\frac{\omega-1}{\omega} & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

6 endpoint flip variants

In[∘]:=
$$\mathtt{flip23} = PD\left[P_{-1,7}, P_{-10,4}, P_{-5,11}, P_{-12,6}, \bar{X}_{-3,2,13,-14}, X_{-8,9,14,-13}\right];$$
$$\mathtt{flip16} = PD\left[P_{-8,2}, P_{-3,9}, P_{-10,4}, P_{-5,11}, \bar{X}_{-1,6,13,-14}, X_{-12,7,14,-13}\right];$$
$$\mathtt{flip34} = PD\left[P_{-1,7}, P_{-8,2}, P_{-5,11}, P_{-12,6}, \bar{X}_{-13,14,4,-3}, X_{-14,13,9,-10}\right];$$
$$\mathtt{flip56} = PD\left[P_{-1,7}, P_{-8,2}, P_{-3,9}, P_{-10,4}, \bar{X}_{-13,14,6,-5}, X_{-14,13,11,-12}\right];$$
$$\mathtt{flip12} = PD\left[P_{-3,9}, P_{-10,4}, P_{-5,11}, P_{-12,6}, \bar{X}_{-13,14,2,-1}, X_{-14,13,7,-8}\right];$$

S = ++--+-, matching (1,6)(2,3)(4,5). Comparing two methods of doing (1,6)(4,5) flip.

In[∘]:=
$$GT_{-5,-12}@GT_{4,11}@GT_{-3,-10}@GT_{-1,-8}@TL@\mathtt{flip23}$$

Out[∘]=

$$0$$

$$\begin{array}{c|cccccccccccc}
 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
 & \gamma_{-12} & \gamma_{6} & \gamma_{-5} & \gamma_{4} & \gamma_{-3} & \gamma_{2} & \gamma_{-1} & \gamma_{7} & \gamma_{-8} & \gamma_{9} & \gamma_{-10} & \gamma_{11} \\
\bar\gamma_{-12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar\gamma_{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar\gamma_{-5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar\gamma_{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar\gamma_{-3} & 0 & 0 & 0 & 0 & 0 & \omega-1 & 0 & 0 & 0 & 1-\omega & 0 & 0 \\
\bar\gamma_{2} & 0 & 0 & 0 & 0 & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 & 0 & 0 & 0 & 0 \\
\bar\gamma_{-1} & 0 & 0 & 0 & 0 & 0 & 1-\omega & 0 & 0 & 0 & \omega-1 & 0 & 0 \\
\bar\gamma_{7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar\gamma_{-8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar\gamma_{9} & 0 & 0 & 0 & 0 & \frac{\omega-1}{\omega} & 0 & -\frac{\omega-1}{\omega} & 0 & 0 & 0 & 0 & 0 \\
\bar\gamma_{-10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar\gamma_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

In[∘]:= $c_{-1,2}$@$c_{-3,6}$@$c_{4,-5}$@$GT_{-5,-12}$@$GT_{4,11}$@$GT_{-3,-10}$@$GT_{-1,-8}$@TL@flip23

Out[∘]=

$$
\begin{array}{c|cccccc}
 & & & 0 & & & \\
 & 1 & 0 & 0 & 0 & -1 & 0 \\
 & (\Upsilon_{-12} & \Upsilon_{7} & \Upsilon_{-8} & \Upsilon_{9} & \Upsilon_{-10} & \Upsilon_{11}) \\
\overline{\Upsilon}_{-12} & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\Upsilon}_{7} & 0 & 0 & \frac{\omega-1}{\omega} & 0 & -\frac{\omega-1}{\omega} & 0 \\
\overline{\Upsilon}_{-8} & 0 & 1-\omega & 0 & \omega-1 & 0 & 0 \\
\overline{\Upsilon}_{9} & 0 & 0 & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 \\
\overline{\Upsilon}_{-10} & 0 & \omega-1 & 0 & 1-\omega & 0 & 0 \\
\overline{\Upsilon}_{11} & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

In[∘]:= $GT_{4,11}$@$GT_{-3,-10}$@$GT_{2,9}$@$GT_{-1,-8}$@TL@flip16

Out[∘]=

$$
\begin{array}{c|cccccccccccc}
 & & & & & & 0 & & & & & & \\
 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\
 & (\Upsilon_{-12} & \Upsilon_{7} & \Upsilon_{-8} & \Upsilon_{9} & \Upsilon_{-10} & \Upsilon_{11} & \Upsilon_{-5} & \Upsilon_{4} & \Upsilon_{-3} & \Upsilon_{2} & \Upsilon_{-1} & \Upsilon_{6}) \\
\overline{\Upsilon}_{-12} & 0 & \omega-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-\omega \\
\overline{\Upsilon}_{7} & -\frac{\omega-1}{\omega} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\omega-1}{\omega} & 0 \\
\overline{\Upsilon}_{-8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\Upsilon}_{9} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\Upsilon}_{-10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\Upsilon}_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\Upsilon}_{-5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\Upsilon}_{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\Upsilon}_{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\Upsilon}_{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\Upsilon}_{-1} & 0 & 1-\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega-1 \\
\overline{\Upsilon}_{6} & \frac{\omega-1}{\omega} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\omega-1}{\omega} & 0
\end{array}
$$

In[∘]:= $c_{-3,6}$@$c_{-1,2}$@$c_{4,-5}$@$GT_{4,11}$@$GT_{-3,-10}$@$GT_{2,9}$@$GT_{-1,-8}$@TL@flip16

Out[∘]=

$$
\begin{array}{c|cccccc}
 & & & 0 & & & \\
 & 1 & 0 & 0 & 0 & -1 & 0 \\
 & (\Upsilon_{-12} & \Upsilon_{7} & \Upsilon_{-8} & \Upsilon_{9} & \Upsilon_{-10} & \Upsilon_{11}) \\
\overline{\Upsilon}_{-12} & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{\Upsilon}_{7} & 0 & 0 & \frac{\omega-1}{\omega} & 0 & -\frac{\omega-1}{\omega} & 0 \\
\overline{\Upsilon}_{-8} & 0 & 1-\omega & 0 & \omega-1 & 0 & 0 \\
\overline{\Upsilon}_{9} & 0 & 0 & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 \\
\overline{\Upsilon}_{-10} & 0 & \omega-1 & 0 & 1-\omega & 0 & 0 \\
\overline{\Upsilon}_{11} & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$